# Time-symmetry, pre- and post-selection, and weak measurements in quantum mechanics

An underlying general principle is that actions or measurements performed in the present can have consequences only **after** they are performed. If the future could have a causal effect on the past within classical physics, then one is immediately led into the famous 'killing-your-grandfather' paradox. Furthermore, if we know the position and velocity $(x, p)_{\mathrm{in}}$ at an initial time $t_{\mathrm{in}}$ along with all the interactions that a classical particle is subjected to, then we can predict with certainty the final state $(x, p)_{\mathrm{fin}}$ at a later time $t_{\mathrm{fin}}$ (see figure 1.a). One may want to perform the measurement at $t_{\mathrm{fin}}$ due to lack of knowledge of the initial state or interactions. But, *in principle*, measurement of *both* the initial and final conditions of a classical system is redundant.

The more subtle situation in quantum mechanics is missed in the standard "time-asymmetric" formulation which inherited our classical tendency to predict the future based on initial conditions. In quantum mechanics, one precisely known initial condition (say a known $|\Psi_{\mathrm{in}}\rangle$), can lead to many possible outcomes for a specific final measurement, say $|\Phi_{\mathrm{fin}}\rangle_1$, $|\Phi_{\mathrm{fin}}\rangle_2$ at $t_{\mathrm{fin}}$. By actually performing subsequent measurements at $t_{\mathrm{fin}}$, we obtain fundamentally new and important information (see figure 1.b). But, we have thus lost our classical ability to predict the future. Einstein decried this consequence of the uncertainty principle saying "God doesn't play dice."

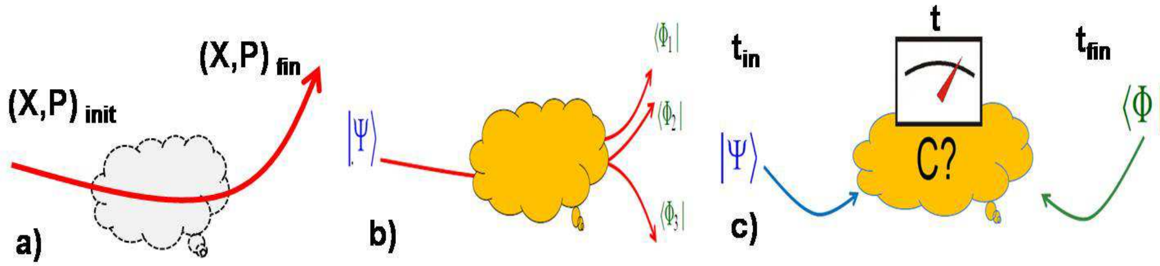


Figure 1: a) Classical scattering experiment, b) quantum scattering experiment, c) ABL 3-phase paradigm

In contrast, Yakir Aharonov and collaborators asked "Why?" Thinking that perhaps the positive in quantum mechanics was hiding behind the negative, they showed that nature gains something very beautiful and exciting by playing dice.

They started with the observation that quantum mechanics does not pick out an arrow, it works just as well from past-to-future as from future-to-past. Then they suggested that the quantum world links the **future with the past** in subtle and significant ways which are often hidden within the quantum uncertainty or noise. Furthermore, they showed that uncertainty is just the right kind to preclude any 'killing-your-grandparent' paradox, i.e. allowing the future to be relevant to the past without violating causality!

The conventional (ideal or strong) measurements can distinguish outcomes, but in the process they disturb the observed system and break these subtle future-to-past links. To measure these links without breaking them, Aharonov introduced 'weak measurements' which allow information to be obtained without causing a disturbance. Aharonov gave a new meaning to quantum uncertainty: it underlies this possibility that outcomes of measurements can also be relevant for the **past** and at the same time allows these future-to-past links to peacefully co-exist with causality, by giving us room to write-off the influence as a mistake.

The weak measurements of interest differ from 'strong' measurements. First, at a time $t_{\mathrm{fin}}$ **after** the weak measurement interaction, one chooses a subset of particles according to the result of a strong measurement performed at $t_{\mathrm{fin}}$. Secondly, the strength of the interaction between the system and measuring device is weak. This means that the pointer in the measuring device is spread out due to quantum uncertainty. The amount that it is 'spread-out,' must be more than the shift given to it by any single weak measurement interaction. This 'spread-out-ness' means that the pointer even has a tail which would correspond to 'impossible' values for the particle.

Suppose the particle was localized around a positive number, so all the interactions with a device move the pointer towards this positive number. Then the experimenter would simply discard any results centered around a negative number as meaningless errors. However, the later post-selection opens the possibility that the future can 'come-back' and select out of the noise rare or apparently impossible properties called weak values, for example, the particle being localized around a negative value.

These properties may represent a new order in physics. They have been successfully verified in numerous experiments. They have proven to be of fundamental importance and have impacted (and we believe will continue to impact) many scientific disciplines as diverse as engineering, condensed matter physics, cosmology and quantum information.

**The Main Idea**

Aharonov, Peter Bergmann and Joel Lebowtiz (ABL) [**?**] re-formulated quantum mechanics in terms of *Pre- and Post-Selected ensembles* (see figure 1.c). ABL contemplated a new measurement paradigm consisting of three stages: a measurement which occurs at the present time $t$ while the state is known both in the past (at time $t_{\text{in}} < t$) (also called a pre-selection) and in the future at $t_{\text{fin}} > t$ (also called a post-selection). (In practice, all three stages are in the past, with $t_{\text{in}} < t < t_{\text{fin}}$.) By collecting only a subset of the outcomes for the later measurement at $t_{\text{fin}}$, the pre-selected-only-ensemble (i.e. $|\Psi_{\text{in}}\rangle$) can be divided into sub-ensembles according to the results of the subsequent post-selection-measurement (i.e. $|\Phi_{\text{fin}}\rangle_1$ or $|\Phi_{\text{fin}}\rangle_2$, etc). Pre-and-post-selected-ensembles are the most refined quantum ensemble, and underly the **T***ime*-**S***ymmetric re-formulation of* **Q***uantum* **M***echanics* (**TSQM**, a/k/a *Two-vector* or **T***wo*-**S***tate*) [**?**].

Because it is a *re-formulation*, experiments cannot prove TSQM over standard quantum mechanics (or vice-versa). To be useful and interesting, any re-formulation should meet several criteria such as those met by TSQM:

1. TSQM is consistent with all the predictions made by standard quantum mechanics (see tab)
2. TSQM revealed strange new features such as Weak Values which can be well outside the eigenvalue spectrum or even be complex. More than a dozen laboratories have already experimentally verified these strange properties by performing Weak Measurements (see tab)
3. TSQM lead to simplifications in calculations and stimulated discoveries in other fields. For example, TSQM led to
    - A new paradigm for precision sensors which have already been used to experimentally probe general physics and even practical/engineering issues, e.g. detecting phenomena previously thought to be un-measureable (see tab below). Signal amplification is of course an immensely important and recurring theme throughout all of science and communication.
    - New mathematics (SuperFourier, see tab)
    - the Quantum Random Walk, a most useful tool in quantum information (see tab)
    - New insights into quantum paradoxes such as the quantum/classical correspondence limit (see tab)
4. Finally, TSQM does suggest experimentally distinguishable generalizations of quantum mechanics (see tab)

How should TSQM and weak values be interpreted? Absent experimental verification of generalizations, one may choose to utilize all the pragmatic, operational advantages listed above, but stick to the standard time-asymmetric formalism. Our view is that these new effects form a logical, consistent, and intuitive pattern and there may be deeper reasons which underly TSQM's success in predicting them. We believe the most natural interpretation suggests that quantum systems be described at a given moment by using two wavefunctions, the standard one evolving from past-towards-the-future and a second one, evolving from future-towards-the-past (see figure 1.c).