3 TSQM lead to new mathematics, simplifications in calculations, and stimulated discoveries in other fields

3.1 Quantum Random Walk

Another fundamental discovery arising out of TSQM is the Quantum-Random-Walk [8] which has also stimulated discoveries in other areas of physics. How can a superposition of small shifts between -1 and 1 give a shift that is arbitrarily far outside ± 1? The answer is that states of the measuring-device interfere constructively for $\hat{P}^{(N)}_{md} = \alpha_w$ and destructively for all other values of $\hat{P}^{(N)}_{md}$ such that $\Phi^{MD}_{fin}(P) \rightarrow \Phi^{MD}_{in}(P - A_w)$, the essence of quantum-random-walk[8]. If the coefficients for a step to the left or right were probabilities, as would be the case in a classical random walk, then N steps of step size 1 could generate an average displacement of $\sqrt{N}$, but never a distance larger than N. However when the steps are superposed with probability *amplitudes*, as with the quantum-random-walk, and when one considers probability amplitudes that are determined by pre-and-post-selection, then the random walk can produce any displacement. In other words, instead of saying that a "quantum step" is made up of probabilities, we say that a quantum step is a superposition of the amplitude for a step "to the left" and the amplitude for a "step to the right," then one can superpose small Fourier components and obtain a large shift. This phenomenon is very general: if $f(t - a_n)$ is a function shifted by small numbers a_n, then a superposition can produce the same function but shifted by a value α well outside the range of a_n: $\sum_{n=0}^{N} c_n f(t - a_n) \approx f(t - \alpha)$. The same values of a_n and c_n are appropriate for a wide class of functions and this relation can be made arbitrarily precise by increasing the number of terms in the sum. The key to this phenomenon is the extremely rapid oscillations in the coefficients $c_n \equiv \frac{(1+\alpha_w)^n (1-\alpha_w)^{N-n}}{2^N} \frac{N!}{n!(N-n)!}$ in $\sum_{n=0}^{N} c_n \exp\left\{ i\lambda \hat{Q}_{md} k_n \right\}$.