Dynamical quantum non-locality

Dynamical non-locality is revealed in the equations of motion. It is fundamentally distinct from kinematic non-locality implicit in quantum correlations. Kinematic non-locality arises from the structure of Hilbert space and does not create any change in probability distributions, causes and effects cannot be distinguished and therefore "action-at-a-distance" cannot manifest. Kinematic non-locality has been extremely useful, having catalyzed, e.g., much of the progress in quantum information science. On the other hand, dynamical non-locality, arises from the structure of the equations of motion and *does* create explicit changes in probability, though in a "causality-preserving" manner.

Dynamical non-locality was first introduced by Aharonov et al in order to explain the nonlocality of topological phenomena such as the Aharonov-Bohm (AB) effect. The AB effect conclusively proved that a magnetic (or electric) field inside a confined region can have a measureable impact on a charged particle which never traveled inside the region. In order to represent the closest correspondence between measurement and theory, Aharonov introduced nonlocal interactions between the particle and field. This was in contrast to the prevailing approach of reifying local interactions with (unphysical) non-gauge invariant quantities outside the confined region, such as the vector (and/or scalar) potential. Both dynamic and kinematic non-locality are generic and can be found in almost every type of quantum phenomenon.

A good example of dynamical nonlocality is the two-slit experiment, the quintessential example of the dual character of quantum mechanics. The initial incoming particle seems to behave as a wave when falling on the (left and right) slits, but when recorded on the screen, its wavefunction "collapses" into that of a localized particle. By repeating the experiment for an ensemble of many particles, the interference pattern manifests through the density of hits along the screen (aligned with, say the x direction): $dn(x)/dx \sim |\psi_{\rm L}(x) + e^{i\alpha}\psi_{\rm R}(x)|^2$ with $\psi_{\rm L}(x)$ coming from the left slit, $\psi_{\rm R}(x)$ from the right (located a distance D away), and α the relative phase between the left and right parts of the wavefunction (see fig. 1).

There are two ways to think about such phenomenon:

The first accepts the Schrödinger description as given, with wavepackets evolving in time. Indeed the Schrödinger description has been extremely useful, having served, for example, as the starting point for the Feynman path integral. The apparent analogy between Schrödinger wave interference and classical wave interference (arising from the use of identical calculations), presents a conceptually simple interpretation of quantum phenomena in terms of our classical picture. One is often advised to apply this consistent formalism for statistical predictions (providing, in this case, probability distributions for the positions of *many* particles) *without* asking questions about it's interpretation. In fact, the belief that the Schrödinger picture is the *only* way by which the interference and relative phase can be inferred, played a central role in the development of the probability amplitude interpretation in the quantum formalism.

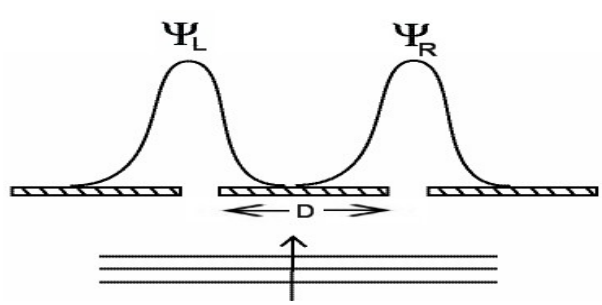

Figure 1: Wavefunction for a single particle in double-slit setup when we do not know through which slit the particle has passed.

The second way of thinking maintains that this is not the end of the story and advocates further inquiry. For example, Feynman [5] stated that such phenomena "..have in it the heart of quantum mechanics. In reality, it contains the only *mystery*." Such proponents often seek to obtain as close a correspondence as possible between theory and measurement. As a consequence, they try to weed out "classical" notions when they have been mis-applied to the quantum realm. For example, classical waves involve many degrees of freedom (e.g. field phenomenon such as sound and electromagnetic waves) and their phase *can* of course be measured by local experiments. But the meaning of a quantum phase is very different. Multiplying the wavefunction $\psi_{\rm L}(x) + e^{i\alpha}\psi_{\rm R}(x)$ by an overall phase ϕ does not change the relative phase α and thus does not yield a different state. Furthermore, it seems that the relative phase α cannot be measured directly on a single particle since it cannot be represented by a Hermitian operator. That is, $\psi_{\rm L}(x) + e^{i\alpha}\psi_{\rm R}(x)$ and $\psi_{\rm L}(x) + e^{i\beta}\psi_{\rm R}(x)$ are not generally orthogonal and thus cannot be eigenstates belonging to different eigenvalues of a Hermitian operator. In further contrast to the classical phase, a change in the relative quantum phase - say from $\psi_{\rm L}(x)+\psi_{\rm R}(x)$ to $\psi_{\rm L}(x)-\psi_{\rm R}(x)$ - would not result in a measureable change in any local properties. The change only shows up in certain non-local properties or much later when the two separate components $\psi_{\rm L}(x)$ and $\psi_{\rm R}(x)$ eventually overlap and interfere. It seems that the relative phase cannot be thought of simply as the difference between a local phase at $\psi_{\rm L}(x)$ and another local phase at $\psi_{\rm R}(x)$.

Another aspect of this second way of thinking is the realization that the Schrödinger wave only has a measureable meaning for an ensemble of particles, not generally for a single particle. This therefore leaves important questions unanswered concerning the *physics* of interference from the perspective of a single particle: if physics obeys local dynamics, then how does the localized particle passing through the right slit sense whether or not the distant left slit is open (closed), causing it to scatter (or not scatter) into a region of destructive interference? Interference experiments have been performed with electron/photon beams whose intensity is sufficiently small such that only one electron/photon traverse the interference apparatus at a time[1] . The interference pattern with light and dark bands is nevertheless built up successively, mark by mark, with each individual "particle-like" electron/photon, [28]. One is then confronted with the fact that a single degree of freedom created the interference pattern. This mystery led Feynman to declare: "Nobody knows how it can be like that." [5]

We follow the second way of thinking and offer a fresh approach to this time honored problem [12] [10] [11] [31] [22] [2] [1]. To motivate the first step, involving a fundamental shift in the types of observables utilized, we make several observations:

First, most discussions of this problem are based on measurements which disturb the interfering particle. This is one of the main reasons that quantum interference is generally considered to be intimately associated with the problems that stem from the statistical character of the quantal description.

Second, the observables studied to date have been simple functions of position and momentum. These observables, however, are **not** sensitive to the relative phase between different "lumps" of the wavefunction (centered around each slit). Nevertheless, the subsequent interference pattern of course is entirely determined by the relative phase between these "lumps," suggesting that simple moments of position and momentum are not the most appropriate dynamical variables to describe quantum interference phenomena.

Third, operators that ***are*** sensitive to the relative phase are exponentials of the position and momentum.

We address the first observation with non-disturbing measurements. To date, several non-disturbing measurements, such as weak measurements and protective measurements, have stimulated lively debates and have proven useful in separating various aspects of quantum theory from the probabilistic aspects [30]. The underlying framework for a new approach to interference [12] [10] [11] [31] [22] [2] [1] is based on another kind of non-disturbing measurement, the "set of deterministic operators" or "deterministic experiments" [10] [22] [31]. This set involves measurement of only those variables for which the state of the system under investigation is an eigenstate. This set answers the question "what is the set of Hermitian operators $\hat{A}_\psi$ for which ψ is an eigenstate?" for any state ψ, i.e. $\hat{A}_\psi = \{\hat{A}_i \textit{ such that } \hat{A}_i|\psi(t)\rangle = a_i|\psi(t)\rangle\,, a_i \in \Re\}$. This question is dual to the more familiar question "what are the eigenstates of a given operator?" Measurement of these operators $\hat{A}_\psi$ does not collapse the wavefunction, since the wavefunction is initially an eigenstate of the operator being measured. Elaboration of this framework is left to existing and forthcoming literature [12] [10] [11] [31]. The essential point needed for this article is the relevance of deterministic experiments for a *single* particle since they can be performed without causing a disturbance.

We address the second and third observations by performing yet another kind of non-disturbing measurement, namely weak measurements, on the observables that *are* sensitive to the relative phase. These observables that are senstive to the relative phase are functions of ***modular variables***. For the case of interference in space, as considered here, the relevant modular variable is modular momentum, not ordinary momentum. These observables are also members of the "deterministic set of operators" and are relevant for an individual particle. We then see that in the context of interference phenomenon, the Heisenberg equations of motion for these modular variables are *non-local*. The nonlocality of these observables is quite intuitive: the operators sensitive to the relative phase simply translate the different "lumps" of the wavefunction. The appropriate translation may cause one lump to overlap with another lump or to overlap simply with the region where the distant slit is

[1] "The most beautiful experiment in physics, according to a poll of *Physics World* readers, is the interference of single electrons in a Youngs double slit." Second place went to Galileos experiment with falling bodies. Third place went to Millikans oil experiment. Fourth went to Newtons prism, etc [41].

either open (or closed). This provides a **physical** explanation for the different behavior of a *single* particle when the distant slit is open or closed. It therefore provides the under-pinnings for a new ontology based on localized particles with non-local interactions, rather than a less physical Schrödinger "wave of probability" traveling throughout all of space.

Prior to APP, dynamical nonlocality was avoided due to the possibility that it could violate causality. However, in a beautiful theorem, APP proved that the dynamical nonlocality they introduced could never violate causality. They considered the general set of conditions necessary to see the non-local exchange of modular variables, for example when the left slit is either monitored or closed and the particle is localized around the right slit. APP proved that these are precisely the same conditions which make the non-local exchange completely uncertain and therefore "un-observable."

While it was beautiful that quantum mechanics allowed "action-at-a-distance" to "peacefully-coexist" with causality, this theorem nevertheless proved to be somewhat anti-climatic: if we cannot actually observe the nonlocal exchange of modular variable, then have we not violated the dictum of maintaining the closest correspondence between measurement and theory by claiming the existence of a new kind of nonlocal - yet un-observable - effect?

Nevertheless, it has been shown that these non-local interactions ***can*** be observed.[1] This has to be done in a causality-preserving manner. Therefore, in order to measure this nonlocality, we must utilize various tools such as pre-selection, post-selection, and weak measurements. Although some of the components utilized in the present analysis were published long ago, they are not generally known and are therefore briefly reviewed.

With this development, we have thereby underscored a fundamental difference between classical mechanics and quantum mechanics that is easily missed from the perspective of the Schrödinger picture: the equations of motion for observables relevant to quantum mechanical interference phenomenon can be **non-local** in a peculiar way that preserves causality. These novel results motivate a new approach to quantum mechanics starting from the Heisenberg picture and involving the set of deterministic operators. While the new framework and associated language are, in principle, equivalent to the Schrödinger formulation, it has led to new insights, new intuitions, new experiments, and even the possibility of new devices that were missed from the old perspective. These types of developments are signatures of a successful re-formulation.

Although further elaboration of this new approach is left to a future article [22], we briefly mention one important conceptual shift: when quantum mechanics is compared to classical mechanics, often the uncertainty or indeterminism of quantum mechanics is emphasized and the profound, fundamental differences in the dynamics is ignored. This is perhaps a result of the similarity between the classical dynamical description (Poisson bracket) and the quantum dynamical description (commutator) for simple functions of momentum or position. Furthermore, uncertainty is viewed in a kind of "negative" light: as a result of the uncertainty in quantum mechanicics, we have lost the ability that we had in classical mechanics to predict the future. Not only is nature "capricious," but it seems that we do not even gain anything from the uncertainty.

The new approach allows us to change this perspective by *deriving* uncertainty from principles that we argue are more fundamental, namely from non-locality and causality. This changes the meaning of uncertainty from one with a "negative" connotation to one with a "positive" connotation. Something similar happened with special relativity when the axioms of relativity were discovered. This inspired a modification of the old language: e.g. that light has the same velocity in all reference frames is certainly highly unusual, but everything works in a self consistent way due to the axiomatic framework, and because of this, special relativity is rather easy to understand.

Similarly, we are convinced that the new approach to interference [12] [10] [11] [31] [22] [2] [1] will lead to a deeper understanding of the nature of quantum mechanics.